Valuing Default and Defeasence Option for Commercial Mortgage Backed Securities

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Abstract

We try to estimate default and deference probabilities of commercial mortgages via an American option pricing framework. In this framework, the borrower is assumed to default either if the price of the real estate drops below the level of the outstanding loan balance or the net operating income of the real estate is less than the periodic payments of the loan. The borrower is assumed to defease when the gain from defeasing to refinance a new project is large enough to cover the loss occurring due to defeasence. To the best of authors' knowledge, this is the first study that simultaneously considers the defeasence and default options of a mortgage borrower.

Key words: Default, Defeasence, Commercial Mortgage, American Option Pricing.

JEL Classification:
1 Introduction

Default and prepayment pricings have long been the focus of mortgage pricing studies. Early studies on this area are mainly focused on residential mortgages. While some studies like Epperson et al. (1985) focused on pricing default on residential mortgages, some other studies like Dunn and McConnell (1981), Brennan Schwartz (1985), Buser and Hendershott (1983) dealt with residential mortgage-backed securities.

Initial attempts on pricing commercial mortgage-backed securities relied very much on residential mortgage literature. Kau et al. (1990) was among the first example modeling the link between the commercial mortgage-backed securities (CMBS) and the underlying mortgage. As residential mortgage studies did, researches on pricing commercial mortgages and mortgage-backed securities depend heavily on option pricing methodologies which model prepayment and default opportunities of mortgage holders as call and put options respectively. Deng et al. (2000) state that the option model does a good job in explaining the default and prepayment. The applicability of this model is highly dependent on their central assumption on the stochastic process that the stock returns are assumed to follow. Here the borrower determines a strategy of either exercising one of these options (namely prepayment and default) or continuing. As Kalotay et al (2004) puts evidence with empirical data the borrowers may act "almost" strategically depending on the value of house prices and interest rates. Furthermore, Deng et al. (2000) argue that the simultaneity of the options is also important in explaining the behavior. But many studies ignore the coexistence of both options and model only one these termination event, either default or prepayment. For example Abraham Theobald (1997) uses the Foster and Van Order’s (1985) single family model to deal with commercial prepayment modelling. (see also Ciochetti and Vandell(1999), Riddiough and Thompson (1993))

Kau et al. (1992) argue that "...contracts with only one of the default and prepayment provisions may lead the borrower to behave differently than when both are present". Since exercising one of these options (namely prepayment and default) excludes exercising the other, simultaneous modeling of prepayment and default is usually called competing risks framework (or models). Ambrose and Sanders (2003) deal with commercial mortgages and commercial mortgage-backed securities in a competing risks model using hazard approach. They mention
the importance of competing risks framework in the commercial mortgage market framework and state that problems arise as adapting contingent claims approach to commercial mortgages.

Besides, Ervolini et al. (1999) and Dierker et al. (2005) also mention that there are important differences between residential mortgage prepayments, and commercial mortgage prepayments: Residential mortgages prepay when interest rates fall (thus present value of future payments increases), however commercial mortgages prepay when interest rates rise.

In a competing risks framework, the conditions under which the borrower exercises his options (default or prepayment) determine the price of those options. There are four main prepayment clauses to reduce or postpone prepayment rates: A prepayment penalty (static or time-varying), yield maintenance, a lock-out period, and defeasance. The first three of these clauses have been searched heavily: Kelly and Slawson (2001) compare the time-varying penalties to others and state that the value of postponing prepayment is often higher for mortgages with declining-rate penalties than for mortgages with static-rate penalties. They advice researchers to consider the effects of time-varying prepayment penalties on the incentive to prepay mortgages. Fu et al.’s (2003) analysis show that while yield maintenance and lockout provisions are relatively more effective than fixed structures in reducing prepayment rates, none of them completely eliminates the risk.

Lefcoe (1999) also mentions prepayment penalties and yield maintenance clauses turn out to be very insufficient to compensate for loss of the commercial mortgage-backed security (CMBS) holders. This could be due to several reasons such as reinvestment losses, loan processing losses, and temporary reductions in interest income. Therefore, defeasance became increasingly common in commercial mortgages. Defeasance practically eliminates both the default risk and prepayment risk of the borrower since defeasance is the exchange of real estate collateral with the Treasury securities. As Dierker et al. (2005) demonstrate, 90% of all commercial mortgages allow prepayment by defeasance. Even though there is a huge literature on prepayment modelling, to the best of our knowledge the unique paper on modelling defeasance belongs to Dierker et al. (2005). Using Longstaff and Schwartz’s (2001) simulation method, they calculate the value of defeasance option and show that the price depends on the rate of return that can be earned on the released equity, current interest rates, and the conditions of the
option contract. However their model lacks default modelling, instead they assume that the short term risky rate is higher than the short term risk free rate by a constant. Concerning the above-mentioned importance of competing risks framework, in this paper, we improve Dierker et al. (2005)’s framework to a competing risks framework by adding a default option. That is, the model assumes that the commercial mortgage holder may terminate his/her mortgage either by default or by defeasance, and values the defeasance and default options of mortgage holder simultaneously. Driessen Van Hemert (2011) study the relative and absolute pricing of CMBX contracts (commercial real estate derivatives) during the recent financial crisis but they neglect the presence of defeasance options in commercial mortgage loans.

This paper tries to model default and defeasance simultaneously. The commercial mortgage holder is assumed to default when either one of the following two conditions met: (i) the price of the real estate he buys is less than the outstanding loan balance (ii) the net operating income of the real estate is less than the periodic payments. The borrower would defease when the value of the real estate or interest rates increases so that the gain from defeasing (to refinance a new project) is large enough from the cost of buying a treasury security. As Christopoulos et al (2008) mention, concerning that the relevant data in this area are either very recent or very costly, we also try use real data in order to compare our results to the prices of the commercial mortgages traded. To the best of our knowledge, this is the first paper in the mortgage pricing literature presenting the numerical results of default and defeasance option under the same model.

The paper proceeds as follows: Section 2 defines the basic concepts, and section 3 models the default and defeasance actions of the borrower. Section 4 gives the valuation results.

2 Basic Concepts

Here we generally follow Dierker et al. (2005)’s model. Assume that the real estate project values $V_t$ is generated from the following stochastic process

$$\frac{dV_t}{V_t} = (r_t + \lambda + \mu - \delta_V) dt + \sigma_V dz_V$$  \hspace{1cm} (1)
where \( \lambda \) is the risk premium that investors demand for holding commercial real estate. The project’s excess return above its required rate of return earned by the developer is denoted by \( \mu > 0 \). The stream of rental income that the owner receives from the property is denoted by \( \delta_V \), and \( \sigma_V \) denotes the volatility of the building value.

Risk-free interest rates in the economy are given by a Vasicek (1977) model, and the dynamics of the short term risk-free rate are described by

\[
    dr_t = \kappa (\alpha - r_t) \, dt + \sigma_r \, dz_r
\]

where \( \kappa \) denotes the speed of mean reversion, \( \alpha \) is the long-run mean short term rate, and \( \sigma_r \) denotes the interest rate volatility. We allow interest rates to be correlated with real estate prices, such that

\[
    E(dz_rdz_V) = \xi dt.
\]

The short term risky rate is given by

\[
    \rho_t = r_t + \delta
\]

where \( \delta > 0 \) denotes the constant credit spread.

We assume that the mortgage has a \( T \)-year amortization schedule with a balloon payment due in year \( \tau, \tau \leq T \), and a fixed monthly contract rate \( r_0 \). We also assume that the project also has an investment horizon of \( \tau \) years.

At time \( t = 0 \) the real estate developer buys one unit of building for \( V_0 \), we assume that \( B_0 \) represents the initial loan, \( K \) represents the down-payment as follows:

\[
    B_0 = LTV_0 \ast V_0
    \]

\[
    K = (1 - LTV_0)V_0
\]
where $LTV_0$ is the loan-to-value ratio at time 0 with $0 < LTV < 1$. If we let $c$ be the fixed monthly payments of the developer, then the loan balance at time $t$ is

$$B_t = \int_t^T e^{-r_0 t} dt = \frac{c}{r_0} \left(1 - e^{-r_0 (T-t)}\right)$$

Let also $NOI$ represent the monthly rental income of the developer from real estate and is assumed to be a certain fraction $\delta_V$ of the real estate’s value $V$, that is $NOI = \delta_V V$.

Debt service coverage ratio is defined accordingly:

$$DSCR = \frac{NOI}{c}.$$

### 3 Default and Defeasance Modelling

We assume that a mortgage borrower has the following defeasance and default opportunities:

1. **Default:** There are two potential attributes in literature that are supposed to trigger commercial mortgage defaults: LTV and DSCR. LTV is related to the mortgage default in the sense that rational mortgagors are supposed to default when the value of the real estate is less than the outstanding loan balance, i.e. LTV is higher than 1. DSCR measures the payment ability of the borrowers and hence the rational borrowers are supposed to default when the NOI is less than the monthly payment. (i.e. DSCR is less than 1). Kau *et al.* (1990) find results supporting the option-theoretic modelling of default which is triggered by the decreasing real estate value. Vandell et al. (1993) finds that LTV is significant in explaining the mortgage default, but the DSCR is not significant. Ambrose and Sanders (2003) find contradictory results and do not find LTV as a significant variable in explaining the default. Ervolini et al. (1999) also suggest that commercial defaults occur when the underlying property’s net operating income ($NOI$) is insufficient to cover debt service, $c$. Combining these ideas and given the above
framework, we model the default as follows: A borrower will default if any of the following conditions met

\[ B_t \geq V_t \]  \hspace{1cm} (3)
\[ c \geq NOI \]  \hspace{1cm} (4)

We could have also expressed those conditions in terms of LTV and DSCR, i.e

\[ LTV \geq 1 \]
\[ DSCR \leq 1 \]

2. Defeasance: Following Dierker et al.(2005) we assume that at time \( s > 0 \) or, equivalently, with remaining term to maturity of \( t \) years where \( t = \tau - s \), we assume that the developer has the American option to defease the loan. Accordingly, the developer may replace the risky commercial mortgage by Treasury securities providing the same payments. Effectively, the developer pays a price of

\[ M^r = \int_0^t ce^{-ur_s(u)} du + Be^{-tr_s(t)} \]

to repay a loan that is only worth

\[ M^\rho = \int_0^t ce^{-ur_s(u)} du + Be^{-tr_s(t)} \]

to non-constrained market participants. Some more explanation about \( r_s \). Since the interest rate of treasury securities is always lower than mortgage rate \( r < \rho \), it follows that \( M^\rho < M^r \), and thus the advantage of defeasance comes from the resultant access to increased liquidity.

Proposition: Assume that the project expires at time \( \tau \) and the loan is defeased at time \( s, s < \tau \). In the absence of default, defeasance allows the developer to take on additional projects with a benefit equal to the constant \( k \) per unit of capital. The constant is given by

\[ k = e^{\mu(\tau - s)} - 1. \]
Since it costs the developer $M^{rs}$ to defease the old loan, the equity position in the project after defeasance is given by $V_s - M^{rs}$. With this down payment, a lender would be willing to lend up to
\[
\frac{LTV_0}{1 - LTV_0} (V_s - M^{rs}). \tag{5}
\]
Consequently, the developer can now invest up to $\frac{1}{1 - LTV_0} (V_s - M^{rs})$ in the project, which is $\frac{LTV_0 \times V_s - M^{rs}}{1 - LTV_0}$ more than the initial investment value. These liquidity benefits of defeasance are then given by
\[
\Pi_s = \frac{LTV_0 \times V_s - M^{rs}}{1 - LTV_0} \times \left( e^{\mu(t-s)} - 1 \right) \tag{6}
\]
Effectively, defeasance provides the developer the option to exchange a portfolio of Treasury securities, worth $M^{rs}$, for the market value of the mortgage $M^{s}_k (< M^{rs})$ plus the liquidity benefits $\Pi_s$ arising from the consequent availability of a subsequent real estate loan.

In the absence of arbitrage opportunities, the value of European option, $D \equiv D(V, r)$ must satisfy the fundamental partial differential equation in the correlated state variables $V$ and $r$. This equation may be solved numerically subject to the exercise condition that the loan is defeased if and only if the benefits of doing so exceeds the costs:
\[
\frac{LTV_0 \times V_s - M^{rs}}{1 - LTV_0} \times \left( e^{\mu(t-s)} - 1 \right) \geq \left( \int_0^t ce^{-ur_s(u)} du + Be^{-tr_s(t)} - \int_0^t ce^{-u\rho_s(u)} du - Be^{-tr_s(t)} \right)
\]

4 Valuation Results

Table 1 and Table 2 tabulate the value of the American option to defease and default under our assumed base-case parameter settings. In particular, we consider an 80% loan-to-value(LtV) ratio ($L = 0.8$), $\tau = 10$ year loan amortized over $T = 30$ years with a $l = 1$ year lockout provision. Using the past 60 months of 3-month U.S. Treasury bill rate data, we parameterize the short-term interest rate process by $\kappa = 0.05$, $\alpha = 0.06$ and $\sigma = 0.01$. The asset value process assumes $\sigma_V = 0.10$ and the correlation between the random changes of the asset price and short-term interest rates is given by $\rho = -0.5$. Finally, the credit spread between risky
and risk-free rates is set at $\delta = 0.1\%$. In this study we are not interested in risk premium of investors, hence set $\lambda = 0$.

From both of the tables, notice that both the value of the option to defease and the option to default depends critically on both the contract rate (going down the rows) as well as the excess rate of return the developer can earn on the released equity (going across the columns). Considering the interest rate model is mean reverting, and the reverting long run mean being set equal to $\alpha = 6\%$ we can classify the three scenarios as follows:

- $r_0 = 2\%$ is the upward sloping term structure case
- $r_0 = 6\%$ is the flat term structure case
- $r_0 = 10\%$ is the downward sloping term structure case

From the borrower’s perspective, a downward sloping term structure (last row of Table 1) is preferable to upward sloping (first row of Table 1) one, since all else equal, in the former case the interest rates are supposed to fall in the future which leads to defease earlier compared to latter case. Therefore the value of the option to defease is higher in the last row. Moving right along columns the excess rate of return that the developer can earn decreases, thus the benefits of defeasance decreases and hence the value of the defeasance option decreases. These results are consistent with Dierker et al. (2005).

<table>
<thead>
<tr>
<th>Defeasance Option</th>
<th>$\mu = 1%$</th>
<th>$\mu = 0.5%$</th>
<th>$\mu = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 2%$</td>
<td>4.1868</td>
<td>1.2339</td>
<td>0.0014</td>
</tr>
<tr>
<td>$r_0 = 6%$</td>
<td>6.2765</td>
<td>2.1104</td>
<td>0.0085</td>
</tr>
<tr>
<td>$r_0 = 10%$</td>
<td>9.2410</td>
<td>3.4763</td>
<td>0.0481</td>
</tr>
</tbody>
</table>

Note: $L = 0.8$, $\sigma_V = 10\%$, $\rho = -0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $\delta = 0.1\%$, $T = 30$ years, $\tau = 10$ years, $l = 1$ years.

In terms of the option to default the scenario is the exact opposite, i.e. in a higher interest rate environment people are less likely to default, so the value of the option to default goes
Table 2: Values of American option to default with NOI and OLB conditions for the base case parameter settings.

<table>
<thead>
<tr>
<th>Default Option</th>
<th>$\mu = 1%$</th>
<th>$\mu = 0.5%$</th>
<th>$\mu = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 2%$</td>
<td>0.0587</td>
<td>0.0776</td>
<td>0.0956</td>
</tr>
<tr>
<td>$r_0 = 6%$</td>
<td>0.0191</td>
<td>0.0256</td>
<td>0.0311</td>
</tr>
<tr>
<td>$r_0 = 10%$</td>
<td>0.0049</td>
<td>0.0050</td>
<td>0.0073</td>
</tr>
</tbody>
</table>

Note: $L = 0.8$, $\sigma_V = 10\%$, $\rho = -0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $\delta = 0.1\%$, $T = 30$ years, $\tau = 10$ years, $l = 1$ years.

down as the interest rates increase, see Table 2. This might seem counterintuitive at first glance.

Across each column in Table 2, one realizes that the excess return is held constant and the shape of the term structure is different. But the overall return rate of the underlying asset value is increasing as we go down the column since the interest rates are getting higher, at least at the beginning of the mortgage period. This initial difference in the asset value can be interpreted as, all else being equal, the initial asset price gets higher as we go down a column.

In option pricing terminology, one can say the distance to default is getting bigger. This makes the default option less likely, and therefore less profitable. It reflects quite clearly in the default option values.

On the other hand, one can argue that when the initial interest rates, $r_0$, (which are used as the contract rates at the same time) rises, this should facilitate the default. Although this is true, the effect of the increase in the asset value due to the higher rate of return ($r_0 + \lambda + \mu - \delta$) dominates this effect. To understand this even better, if we analyze Table 2 across a row, we observe that even a small decrease in the excess return of the underlying asset has a significant positive impact on the default option value. As the excess return gets small, the distance to default gets small and therefore the default is more likely and the option price is significantly higher.

With regard to $\mu$ and the value of the defeasance option, the higher the excess rate of return that the developer can earn, all else equal, the greater the benefits of the defeasance and so the more valuable this option is.
We can imagine a lower boundary for default and an upper boundary for defeasance for the asset price process. Namely, a borrower decides to exercise her defeasance option if the asset price hits the upper boundary, and decides to exercise the default option if the asset price hits the lower boundary. Hence, a general rule of thumb can be deduced as the higher the asset price, the higher the defeasance option value. And accordingly, the lower the asset price the higher the default option value. This observation is verified in almost all of our sensitivity analysis tables. The defeasance and default option prices always move in opposite directions. This observation can also be interpreted in terms of a cost-benefit analysis for the options, e.g. the asset price is the benefit of the defeasance but the cost of the default option.

On the other hand, with regard to the relationship between the initial interest rates (contract rates) and the value of the defeasance option can be read through the payoff of the option, see equation (5). The payoff is proportional to the difference between the current asset value and the current bond price (with equivalent cash flow with the mortgage contract). Higher interest rate make the bond price lower, therefore the difference higher. As a result of this, we observe the increasing values in the defeasance option price as we go down each column, i.e. as the interest rates increase.

5 Sensitivity Analysis

Table 3: Values of American option to defease and option to default for the positive correlation between the asset price and the interest rate.

<table>
<thead>
<tr>
<th>Defeasance Option</th>
<th>$\mu = 1%$</th>
<th>$\mu = 0.5%$</th>
<th>$\mu = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 2%$</td>
<td>4.6341</td>
<td>1.5171</td>
<td>0.0129</td>
</tr>
<tr>
<td>$r_0 = 6%$</td>
<td>6.6288</td>
<td>2.3467</td>
<td>0.0359</td>
</tr>
<tr>
<td>$r_0 = 10%$</td>
<td>9.4658</td>
<td>3.6370</td>
<td>0.1022</td>
</tr>
</tbody>
</table>

Note: $L = 0.8$, $\sigma_V = 10\%$, $\rho = 0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $\delta = 1\%$, $T = 30$ years, $\tau = 10$ years, $l = 5$ years.

In Table 3 and 4, we hold all other parameter values unchanged but now assume that random changes in the asset price and the random changes in the interest rate are positively correlated, $\rho = 0.5$. This situation is a bit counter intuitive, however it is case that is worth looking at. As expected, values of the option to defease increase compared to Table 1. Intu-
Table 4: Values of American option to default with NOI and OLB conditions for the positive correlation between the asset price and the interest rate.

<table>
<thead>
<tr>
<th>Default Option</th>
<th>$\mu = 1%$</th>
<th>$\mu = 0.5%$</th>
<th>$\mu = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 2%$</td>
<td>0.1305</td>
<td>0.1657</td>
<td>0.1982</td>
</tr>
<tr>
<td>$r_0 = 6%$</td>
<td>0.0455</td>
<td>0.0577</td>
<td>0.0774</td>
</tr>
<tr>
<td>$r_0 = 10%$</td>
<td>0.0093</td>
<td>0.0137</td>
<td>0.0176</td>
</tr>
</tbody>
</table>

Note: $L = 0.8$, $\sigma_V = 10\%$, $\rho = 0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $\delta = 1\%$, $T = 30$ years, $\tau = 10$ years, $l = 5$ years.

Itively, higher building values, when the benefits of defeasance are correspondingly greater, are associated with higher interest rates, which in turn leads to lower cost of defeasance, therefore higher values of defeasance options. For example, under our base-case assumptions and for $r_0 = 2\%$ and $\mu = 0.5\%$, the value of the option to defease equals 1.2339% of the mortgage’s face value at origination, but increases to 1.5171% when the random changes are positively correlated. By the same token, higher building values, when the cost of the default is higher, are associated with higher interest rates, which leads to smaller benefit of the option, i.e. smaller net present value of the outstanding loan balance. For the same set of parameters the value of the option to default equals 0.0766% of the mortgage’s face value at origination, but increases to 0.1657%.

The Tables 5-6 and the Tables 7-8 investigate the sensitivities of defeasance and default option values to increases in underlying volatilities. In Tables 5-6 we double interest rate volatility from $\sigma_r = 0.1$ to $\sigma_r = 0.2$, while in Table 7-8 we double asset volatility from $\sigma_V = 0.10$ to $\sigma_V = 0.20$. In both cases the values of the option to defease and default increase compared to the base-case scenario. Increases in volatility, either the volatility of interest rates or the volatility of asset price, increase option values. This is in line with our usual intuition about the connection between the option price and the volatility. Regardless of the type of option, option prices are often increasing functions of volatility. We merely demonstrate this general property of the options in this setting.
Table 5: Values of American option to defease for the increased interest rate volatility.

<table>
<thead>
<tr>
<th>Defeasance Option</th>
<th>(\mu = 1%)</th>
<th>(\mu = 0.5%)</th>
<th>(\mu = 0.1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_0 = 2%)</td>
<td>5.2029</td>
<td>1.8566</td>
<td>0.0398</td>
</tr>
<tr>
<td>(r_0 = 6%)</td>
<td>7.1430</td>
<td>2.6735</td>
<td>0.0801</td>
</tr>
<tr>
<td>(r_0 = 10%)</td>
<td>9.8687</td>
<td>3.9140</td>
<td>0.1688</td>
</tr>
</tbody>
</table>

Note: \(L = 0.8, \sigma_V = 10\%, \rho = -0.5, \kappa = 0.05, \alpha = 0.06, \sigma_r = 2\%, \delta = 1\%, T = 30\) years, \(\tau = 10\) years, \(l = 5\) years.

Table 6: Values of American option to default with NOI and OLB conditions for the increased interest rate volatility.

<table>
<thead>
<tr>
<th>Default Option</th>
<th>(\mu = 1%)</th>
<th>(\mu = 0.5%)</th>
<th>(\mu = 0.1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_0 = 2%)</td>
<td>0.0582</td>
<td>0.0813</td>
<td>0.1011</td>
</tr>
<tr>
<td>(r_0 = 6%)</td>
<td>0.0188</td>
<td>0.0254</td>
<td>0.0306</td>
</tr>
<tr>
<td>(r_0 = 10%)</td>
<td>0.0036</td>
<td>0.0051</td>
<td>0.0063</td>
</tr>
</tbody>
</table>

Note: \(L = 0.8, \sigma_V = 10\%, \rho = -0.5, \kappa = 0.05, \alpha = 0.06, \sigma_r = 2\%, \delta = 1\%, T = 30\) years, \(\tau = 10\) years, \(l = 5\) years.

Table 7: Values of American option to defease for the increased asset volatility.

<table>
<thead>
<tr>
<th>Defeasance Option</th>
<th>(\mu = 1%)</th>
<th>(\mu = 0.5%)</th>
<th>(\mu = 0.1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_0 = 2%)</td>
<td>6.4841</td>
<td>2.5653</td>
<td>0.01347</td>
</tr>
<tr>
<td>(r_0 = 6%)</td>
<td>8.4197</td>
<td>3.4247</td>
<td>0.2095</td>
</tr>
<tr>
<td>(r_0 = 10%)</td>
<td>11.0898</td>
<td>4.6455</td>
<td>0.3371</td>
</tr>
</tbody>
</table>

Note: \(L = 0.8, \sigma_V = 20\%, \rho = -0.5, \kappa = 0.05, \alpha = 0.06, \sigma_r = 1\%, \delta = 1\%, T = 30\) years, \(\tau = 10\) years, \(l = 5\) years.

Table 8: Values of American option to default with NOI and OLB conditions for the increased asset volatility.

<table>
<thead>
<tr>
<th>Default Option</th>
<th>(\mu = 1%)</th>
<th>(\mu = 0.5%)</th>
<th>(\mu = 0.1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_0 = 2%)</td>
<td>2.6444</td>
<td>2.8535</td>
<td>3.0095</td>
</tr>
<tr>
<td>(r_0 = 6%)</td>
<td>1.9273</td>
<td>2.0947</td>
<td>2.2324</td>
</tr>
<tr>
<td>(r_0 = 10%)</td>
<td>1.1879</td>
<td>1.2893</td>
<td>1.4287</td>
</tr>
</tbody>
</table>

Note: \(L = 0.8, \sigma_V = 20\%, \rho = -0.5, \kappa = 0.05, \alpha = 0.06, \sigma_r = 1\%, \delta = 1\%, T = 30\) years, \(\tau = 10\) years, \(l = 5\) years.
Table 9 and Table 10 tabulate the value of the American option to defease and to default where the loan-to-value ratio is decreased by 10%. We hold all other parameter values unchanged but now assume that the maximum LtV ratio is 70% instead of 80%. Here we compare the results with Table 1 and 2. Notice that with a lower loan-to-value ratio, the values of the option to defease decrease. The intuition here follows from the fact that with a lower loan-to-value ratio, all else equal, the developer cannot lever the released equity by as much, thereby decreasing the benefits of defeasance. On the other hand, the value of the option to default is severely decreased, simply because of the fact that the amount that the developer borrows is decreased. In turn, this decreases the probability of default, which automatically makes the value of the default option drop down.

Table 9: Values of American option to defease for a lower loan-to-value ratio.

<table>
<thead>
<tr>
<th>Defeasance Option</th>
<th>$\mu = 1%$</th>
<th>$\mu = 0.5%$</th>
<th>$\mu = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 2%$</td>
<td>2.6342</td>
<td>0.8222</td>
<td>0.0063</td>
</tr>
<tr>
<td>$r_0 = 6%$</td>
<td>3.7208</td>
<td>1.2606</td>
<td>0.0146</td>
</tr>
<tr>
<td>$r_0 = 10%$</td>
<td>5.3304</td>
<td>1.9402</td>
<td>0.0383</td>
</tr>
</tbody>
</table>

Note: $L = 0.7$, $\sigma_V = 10\%$, $\rho = -0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $\delta = 1\%$, $T = 30$ years, $\tau = 10$ years, $l = 5$ years.

Table 10: Values of American option to default with NOI and OLB conditions for a lower loan-to-value ratio.

<table>
<thead>
<tr>
<th>Default Option</th>
<th>$\mu = 1%$</th>
<th>$\mu = 0.5%$</th>
<th>$\mu = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 2%$</td>
<td>0.0062</td>
<td>0.0091</td>
<td>0.0125</td>
</tr>
<tr>
<td>$r_0 = 6%$</td>
<td>0.0016</td>
<td>0.0021</td>
<td>0.0033</td>
</tr>
<tr>
<td>$r_0 = 10%$</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Note: $L = 0.7$, $\sigma_V = 10\%$, $\rho = -0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $\delta = 1\%$, $T = 30$ years, $\tau = 10$ years, $l = 5$ years.

Table 11 and Table 12 tabulate the value of the American option to defease and to default where the credibility of the investor is higher. Notice that a 50bps decrease in the credit spread enhances option values of defeasance by reducing the costs of the cash flow replicating portfolio. For example, for $r_0 = 2\%$ and $\mu = 0.5\%$, under our base-case assumptions the value of the option to defease equals 1.2339% of the mortgage’s face value at origination, but now increases to 1.9505%. However, it does not have a significant effect on the value of the option.
to default, since we assume that the credibility of the developer remains the same throughout the whole time.

Table 11: Values of American option to defease for a lower credit spread.

<table>
<thead>
<tr>
<th>Defeasance Option</th>
<th>$\mu = 1%$</th>
<th>$\mu = 0.5%$</th>
<th>$\mu = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 2%$</td>
<td>5.0754</td>
<td>1.9505</td>
<td>0.0603</td>
</tr>
<tr>
<td>$r_0 = 6%$</td>
<td>7.1707</td>
<td>2.9068</td>
<td>0.1392</td>
</tr>
<tr>
<td>$r_0 = 10%$</td>
<td>10.0875</td>
<td>4.2851</td>
<td>0.3107</td>
</tr>
</tbody>
</table>

*Note: $L = 0.8$, $\sigma_V = 10\%$, $\rho = 0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $\delta = 0.5\%$, $T = 30$ years, $\tau = 10$ years, $l = 5$ years.*

Table 12: Values of American option to default with NOI and OLB conditions for a lower credit spread.

<table>
<thead>
<tr>
<th>Default Option</th>
<th>$\mu = 1%$</th>
<th>$\mu = 0.5%$</th>
<th>$\mu = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 2%$</td>
<td>0.0600</td>
<td>0.0765</td>
<td>0.0982</td>
</tr>
<tr>
<td>$r_0 = 6%$</td>
<td>0.0181</td>
<td>0.0246</td>
<td>0.0315</td>
</tr>
<tr>
<td>$r_0 = 10%$</td>
<td>0.0038</td>
<td>0.0051</td>
<td>0.0062</td>
</tr>
</tbody>
</table>

*Note: $L = 0.8$, $\sigma_V = 10\%$, $\rho = 0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $\delta = 0.5\%$, $T = 30$ years, $\tau = 10$ years, $l = 5$ years.*

6 Empirical Results

6.1 Data

We obtained the CMBS loan data from Trepp’s comprehensive historical commercial loan database. The data consists of three pools two of which has AA rating, one with BA rating. Further detailed description and analysis of the different credit pools in the data has been provided in the implementation section, section 5.3. The data consists of 310 different fixed rate CMBS loans observed at monthly intervals from September 2000 to March 2005. All loans have balloon payments in their cash flow structure.

However, we did not have the specific price data for the underlying real estate in each of these loans. Hence, we have used the REIT index as a proxy for the real estate price for all the loans. We obtained the REIT index from Bloomberg. Also from Bloomberg, we used the
10 year US treasury rate as a proxy for the current interest rate in the corresponding fixture of dates that the REIT index is announced.

The parameters that appears in equations (1) and (2) have been jointly estimated using an iterative optimization procedure, including the correlation parameter $\rho$ between the random noise parts of the two stochastic processes. Only the $\delta$ parameter that reflects the credit rating of the borrower has been obtained from the CMBS loan data.

### 6.2 Case Analysis

In addition to all the descriptive statistics studies that have been explored in section 4, we would like to add one last analysis to the pile since it would shed some light on the result that we obtain in section 5.3 when we implement our model with real data. The results indicate the most important two components of the model are the loan-to-value ratio, $L$, and the credit rating parameter, $\delta$ when it comes to find the value of the defeasance and default options in CMBS loan deals.

<table>
<thead>
<tr>
<th>Defeasance Option</th>
<th>$\delta = 1%$</th>
<th>$\delta = 0.5%$</th>
<th>$\delta = 0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 90%$</td>
<td>16.0521</td>
<td>17.0775</td>
<td>17.8715</td>
</tr>
<tr>
<td>$L = 80%$</td>
<td>6.2469</td>
<td>7.1297</td>
<td>7.8494</td>
</tr>
<tr>
<td>$L = 70%$</td>
<td>3.1945</td>
<td>3.9001</td>
<td>4.5309</td>
</tr>
</tbody>
</table>

*Note:* $\mu = 1\%$, $\sigma_V = 10\%$, $\rho = 0.5$, $\kappa = 0.05$, $\alpha = 0.06$, $\sigma_r = 1\%$, $r_0 = 6\%$, $T = 30$ years, $\tau = 10$ years, $l = 5$ years.

Let’s focus on Table 13 first. In valuation of the defeasance option both LTV and credit rating is important, however LTV is the clear winner in this case in terms of its impact on the price of the option.

Some more discussion on the effect of $\delta$. So when we compare two different loans with different credit ratings it is going matter too much as long as they have similar LTV ratios. This
Table 14: Values of American option to defease and option to default for different \textbf{loan-to-value} ratios and for different \textbf{credit ratings}.

<table>
<thead>
<tr>
<th>Default Option</th>
<th>$\delta = 2%$</th>
<th>$\delta = 1%$</th>
<th>$\delta = 0.5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 90%$</td>
<td>0.1881</td>
<td>0.1557</td>
<td>0.1320</td>
</tr>
<tr>
<td>$L = 80%$</td>
<td>0.0209</td>
<td>0.0167</td>
<td>0.0121</td>
</tr>
<tr>
<td>$L = 70%$</td>
<td>0.0016</td>
<td>0.0013</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

\textit{Note: $\mu = 1\%, \sigma_V = 10\%, \rho = 0.5, \kappa = 0.05, \alpha = 0.06, \sigma_r = 1\%, r_0 = 6\%, T = 30$ years, $\tau = 10$ years, $l = 5$ years.}

observation will be confirmed by our empirical study in the next section.

On the other hand, Table 14 suggests that both LTV and credit rating is important in terms of default option pricing, and the winner is rather unclear in this case. As the LTV goes down, or credit rating goes up, i.e. the loan becomes more credit worthy, the value of the default option goes down as expected. And both components have a fair share of their contribution in the price. This observation will again be confirmed in next section.

\section*{6.3 Implementation of the Model}

Here we calibrate the model with the real data. The data consists of three pools two of which has AA rating, one of which has BA rating. The following figures presents the average default and defeasance premiums of the pools through time. (The average default and defeasance premium indicates the average prices of the loans in the pool.)

The proper way to distinguish each of these pools from each other would be via their LTV ratios and credit ratings. Obviously, the credit ratings of both AA rating loans are the same (or very similar), yet their LTV ratios are quite different due to the type of the property underlying the mortgage. 1301AA rated loans have significantly lower LTV’s than 1166AA loans. Figure 1 provides the time series of default premium results. In Figure 1, the average default price of the BA rated pool is always higher than the AA rated pool and the spread remains constant over the course of the mortgage. This is in line with what we expect since the pool with the lower rating has a higher probability of default and thus higher default option price. But the average prices of the AA rated pools, namely 1301, 1166, do not a have spread in
between them. This may be due to the similar characteristic of the pools since they have the similar rating. It is also worth to note that the average of the prices two AA rated pools is also higher than the BA rated pools.

Figure 2: Defeasance Option Price comparison for different credit pools
Figure 2 reveals an interesting implication of our setting. Note that the BA rated loans have worse credit rating than 1166AA rated loans, however as we found out in section 5.2 when it comes to the valuation of the defeasance option LTV is the main component to look at, rather than the credit rating. This is evident in figure 2, where two different types of loans, namely BA001 and 1166AA have similar valuation since their LTV’s are quite close to each other and significantly higher than 1301AA rated loans.

7 Conclusion

Defeasance and default can be thought as two different ways of early termination of a mortgage contract. In case of the defeasance the borrower exchanges the US Treasury bonds and his future obligations. However, his benefits do not end there since also he benefits from having been able to use the accumulated sum of money for a future investment. Likewise for the case of default, the borrower gives up the real estate, and gets rid of his future obligations in return. Also here the borrower decides to default when the action maximizes the value of his position. Therefore, valuation of both the defeasance and the default can be considered as an American option pricing problem.

Even though the model proposed in the paper looks quite simple, it enlightens the role of quite a few different parameters in the valuation of defeasance and default options on a commercial loan. We confirm a few findings that exist already in the literature, such as the residential mortgages are prepaid when the interest rates fall but commercial mortgages defease when the interest rates are high, and therefore the cost of defeasance is low; or the value of the option to defease do not only depend on the interest rate but also on the rate of return that the borrower could generate on his investment.

In addition to these the model also suggests that in valuation of both the defeasance and the default option both LTV ratios and credit ratings play a crucial role, however the impact of high LTV ratio dominates the impact of low credit rating. On the other hand they both play important roles in valuation of the default option.
8 Acknowledgements

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References


